

# A MULTIGRID METHOD FOR UNFITTED FINITE ELEMENT METHODS

Sven Gross<sup>1</sup>, Thomas Ludescher<sup>1,\*</sup>, Arnold Reusken<sup>1</sup>

<sup>1</sup> Chair of Numerical Mathematics, RWTH Aachen University  
Templergraben 55, 52056 Aachen, Germany  
{gross,ludescher,reusken}@igpm.rwth-aachen.de  
www.igpm.rwth-aachen.de/team/{gross,ludescher,reusken}

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For the numerical treatment of interface problems, such as the fluid dynamics in two-phase flows it is important to capture the discontinuous behavior of the solution at the interface in order to obtain a good approximation quality and optimal convergence rates. Discretizations of such interface problems with standard finite element methods on a mesh that is *not fitted* to the interface do not yield satisfactory results. In unfitted FEM (also called XFEM or CutFem) the solution space is enriched by a set of cut standard basis functions close to the interface, which then allows an accurate representation of discontinuities. Typically, the arising linear systems are very ill-conditioned when the interface is close to an element node and hence the design of an efficient linear solver becomes a challenging task.

In this talk we present a multigrid solver for the efficient solution of this type of unfitted finite element discretizations. One important component of multigrid methods is transfer of information between different grid levels. For that purpose suitable prolongation and restriction operators need to be designed. Standard techniques cannot be applied for unfitted finite element discretizations. We present the construction of an appropriate transfer operator.

The method will be explained and results of several numerical experiments, e.g. for a Poisson interface problem using a Nitsche discretization [1], will be presented. Also the robustness of the multigrid solver with respect to the size of the jump in the diffusion coefficient will be addressed.

## REFERENCES

- [1] Hansbo, A. and Hansbo, P. An unfitted finite element method based on Nitsche's method for elliptic interface problems. *Comput.Methods in Appl.Mech.Eng.* (2002) **191**(47), pp. 5537-5552