

# The Discontinuous Galerkin Material Point Method for the simulation of hyperbolic problems in finite deforming solids.

A. Renaud<sup>\*1</sup>, T. Heuzé<sup>1</sup> and L. Stainier<sup>1</sup>

<sup>1</sup> Ecole Centrale de Nantes, 1 Rue de la Noë, 44321 Nantes, France  
{adrien.renaud,thomas.heuze,laurent.stainier}@ec-nantes.fr

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A wide variety of dynamic physical problems, such as impact on structures or high-speed forming techniques, involves waves propagating in finite strained solids. The numerical simulation of this class of problems has been and is still mainly performed with the Finite Element Method despite well-known short-comings. Indeed, Lagrangian formulations of FEM may suffer from mesh entanglement and classical time integrators introduce high frequency noise in the vicinity of discontinuous solutions. Two families of numerical methods developed in parallel address each of these limitations. First, among Particle-In-Cell methods, the Material Point Method [1] can be viewed as the classical FEM with moving quadrature points and is an effective tool for dealing with finite deformations due to the use of particles that can move in an arbitrary Eulerian grid. However, this method inherits the FEM spurious oscillations issues. Second, the Discontinuous Galerkin approximation [2], based on discontinuous shape functions across element boundaries, allows to accurately track waves fronts within a finite element formulation. Nevertheless, the DGFEM is also subjected to mesh tangling effects.

The purpose of this work is the extension of the MPM to the DG framework in order to deal with both continuous and discontinuous waves in solids undergoing finite deformations. The resulting Discontinuous Galerkin Material Point Method uses the same space discretization than that of the MPM with an arbitrary computational grid supporting the DG approximation. Interface fluxes arising in the weak form of a system of conservation laws, written element-by-element on the reference configuration, are computed by means of an approximate Riemann solver. A von Neumann stability analysis shows that the DGMPM can be used with a less restrictive Courant condition compared to that of the MPM and the DGFEM. In particular, the optimal CFL condition is possible for a special space discretization. This method is illustrated and compared to the original MPM, an analytical solution and classical FEM on one, and two-dimensional problems.

## REFERENCES

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