

# A COMBINED $rh$ -ADAPTIVE STRATEGY FOR THE LOCAL MAXIMUM ENTROPY POINT COLLOCATION METHOD

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Most numerical methods in solid mechanics work with discretised weak-forms. Point collocation methods work with discretised strong-forms and have received much less study. Adaptivity procedures have been used for many years to improve the efficiency of weak-form based methods. For instance  $r$ -adaptivity in [1] is for finite element methods where the material residual forces were used as the indicators to relocate the mesh and mesh optimization was required to avoid mesh distortion. Alternatively in  $h$ -adaptivity for mesh based methods, the error from the current solution is estimated to determine the regions that need to be further refined. These can be combined to give  $rh$ -adaptivity as in [2] where  $rh$ -adaptivity is shown to improve weak-form based methods more efficiently than a pure  $r$  or  $h$ -adaptivity.

In this paper, a combined  $rh$ -adaptive strategy is proposed for the local maximum entropy point collocation method (MEPCM) for linear elasticity problems. Point collocation methods are strong-form based methods, which present some attractive features, e.g. no background mesh, as compared to the weak-form based methods. In this implementation of  $r$ -adaptivity, the collocation points are relocated iteratively to achieve material force balance. Since the material residual forces are derived at the collocation points, mesh distortion is eliminated in the MEPCM and the relocation process is simplified without the mesh optimization. In  $h$ -adaptivity, the strong-form based formulation is used to evaluate the current solution and new points are inserted in the regions with high local errors. A pure  $r$  and a pure  $h$ -adaptivity are combined to find optimal positions for points in which the number of points is minimal for a specified accuracy. Several numerical examples are demonstrated to validate the behaviour of this  $rh$ -adaptivity in the MEPCM and comparisons are made of convergence rates and computational cost for uniform refinement, pure  $r$ -adaptivity, pure  $h$ -adaptivity and combined  $rh$ -adaptivity.

## REFERENCES

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