

AN H/P-ADAPTIVE ENTROPY STABLE SUMMATION-BY-PARTS SCHEME FOR THE EULER AND NAVIER-STOKES EQUATIONS ON CURVILINEAR GRIDS

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ABSTRACT

In this talk, I will discuss two approaches to distribute the degrees of freedom for schemes of arbitrary order: 1) h-refinement/coarsening where elements are subdivided or agglomerated, and 2) p-refinement/coarsening where the degree of the discretization within an element is increased or decreased. The emphasis will be on the solution of the compressible Euler and Navier-Stokes (NS) equations in complex geometries, however, the presented methodology is general. We achieve robustness by constructing methods that are provably stable (under appropriate assumptions); the interest here is in methods that only allow growth in time as dictated by the continuous problem. Both the Euler and NS equations are equipped with an incomplete L2 stability theory— in that positivity of density and temperature are assumed — in the form of a secondary conservation law (for smooth flows), the conservation of entropy. Thus, L2 stability at the continuous level is proven by showing that entropy is conserved or dissipated (smooth/discontinuous solutions). We concentrate on schemes with semi-discrete stability proofs that follow the continuous proofs in a one-to-one manner. To do so, we construct schemes equipped with the summation-by-parts (SBP) property, which demands that the scheme mimic, to high order, integration by parts. L2 stability is accomplished by combining the SBP property with two point entropy conservative fluxes; the resulting stability proof is sharp and does not require the assumption of integral exactness. This combination has previously been used to construct entropy stable schemes that allow for h/p adaptivity on Cartesian grids. I will discuss how to extend these ideas to body-fitted curvilinear meshes necessary for real-world applications on complex geometries.