

## DEVELOPMENT OF A NEW CONSISTENT DISCRETE GREEN OPERATOR FOR FFT-BASED METHODS TO SOLVE HETEROGENEOUS PROBLEMS WITH EIGENSTRAINS

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A new expression of the periodized Green operator using the Discrete Fourier Transform method and consistent with the Fourier grid is derived from the classic “Continuous Green Operator (CGO)”, in order to avoid the problem referred to as aliasing inherent to Discrete Fourier Transform methods. It is shown that the easy use of the conventional continuous Fourier transform of the modified Green Operator for heterogeneous materials with eigenstrains leads to spurious oscillations when computing the local responses of composite materials close to materials discontinuities like interfaces, dislocations, edges. In this work, we also focus on the calculation of the displacement field and its associated discrete Green operator which may be useful for materials characterization methods (diffraction, etc...). We show that the development of these new consistent discrete Green operators in the Fourier space named “Discrete Green Operators” (DGO) allows to eliminate oscillations while retaining better convergence capability. For illustration, a DGO for strain-based modified Green tensor is implemented in a fixed-point algorithm for heterogeneous periodic composites known as the Moulinec and Suquet “basic scheme” ([1,2]) that is extended here to consider eigenstrain fields, as in Anglin [3].

Numerical examples are reported such as the computation of the local stresses and displacement of composite materials with homogeneous or heterogeneous elasticity combined to dilatational eigenstrain or eigenstrain representing prismatic dislocation loops. The numerical stress and displacement (Figure) solutions obtained with the DGO are calculated for cubic-shaped inclusions, spherical Eshelby and inhomogeneity problems. The results are discussed and compared with analytical solutions [4,5], and classic discretization method using the CGO [5].

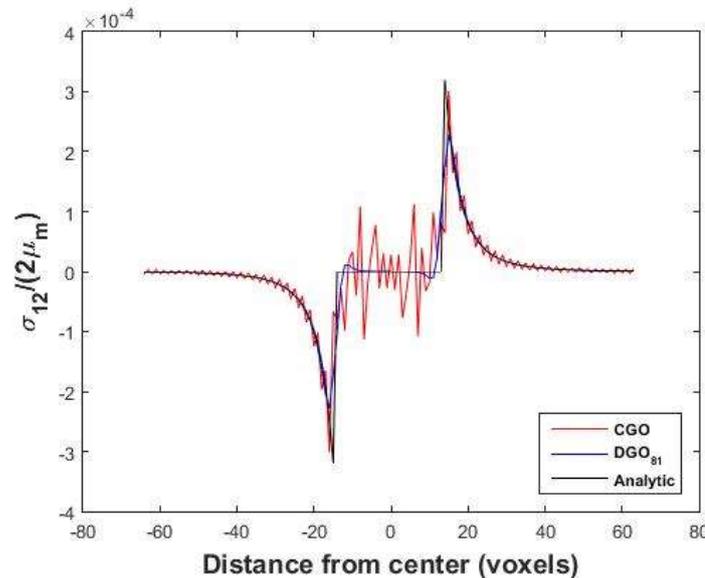


Figure : Stress field component  $\sigma_{12}/(2*\mu)$  along a line parallel to the x-axis through the center of the unit cell due to a heterogeneous spherical inclusion containing a dilatational eigenstrain  $\epsilon_{ij}^*$  ( $\epsilon_{ij}^* = \mathbf{0}$  except  $\epsilon_{11}^* = \epsilon_{22}^* = \epsilon_{33}^* = \mathbf{0.005}$ ). Analytical solution, FFT-based solution with continuous Green operator CGO and discret Green operator DGO. Calculation with  $128 * 128 * 128$  voxels

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